SEMINARIO

TRANSDISCIPLINARIO

CINVESTAV

11 JUNIO 2012

Contar

$$Q(t) = \text{Cantidad al tiempo } t$$

$$Q(t+h) = Q(t) + N[t, t+h] - M[t, t+h] +$$
$$+ E[t, t+h] - S[t, t+h]$$

$$\text{Si} \quad E[t, t+h] = 0 \;\; \text{y} \;\; S[t, t+h] = 0$$

$$Q(t+h) = Q(t) + N[t, t+h] - M[t, t+h]$$

$$Q(t+h) = Q(t) + \beta h Q(t) - \mu h Q(t)$$

Ecuación en diferencias

Contar

$$Q(t+h)-Q(t)=(\beta-\mu)hQ(t)$$

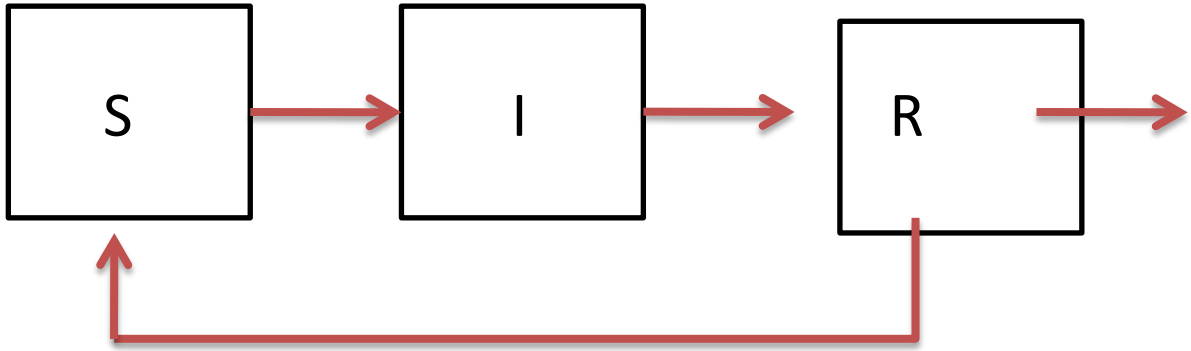
Haciendo $\quad r=(\beta-\mu)$

$$\frac{Q(t+h)-Q(t)}{h} = rQ(t)$$

Bajo la suposición de muestras muy grandes

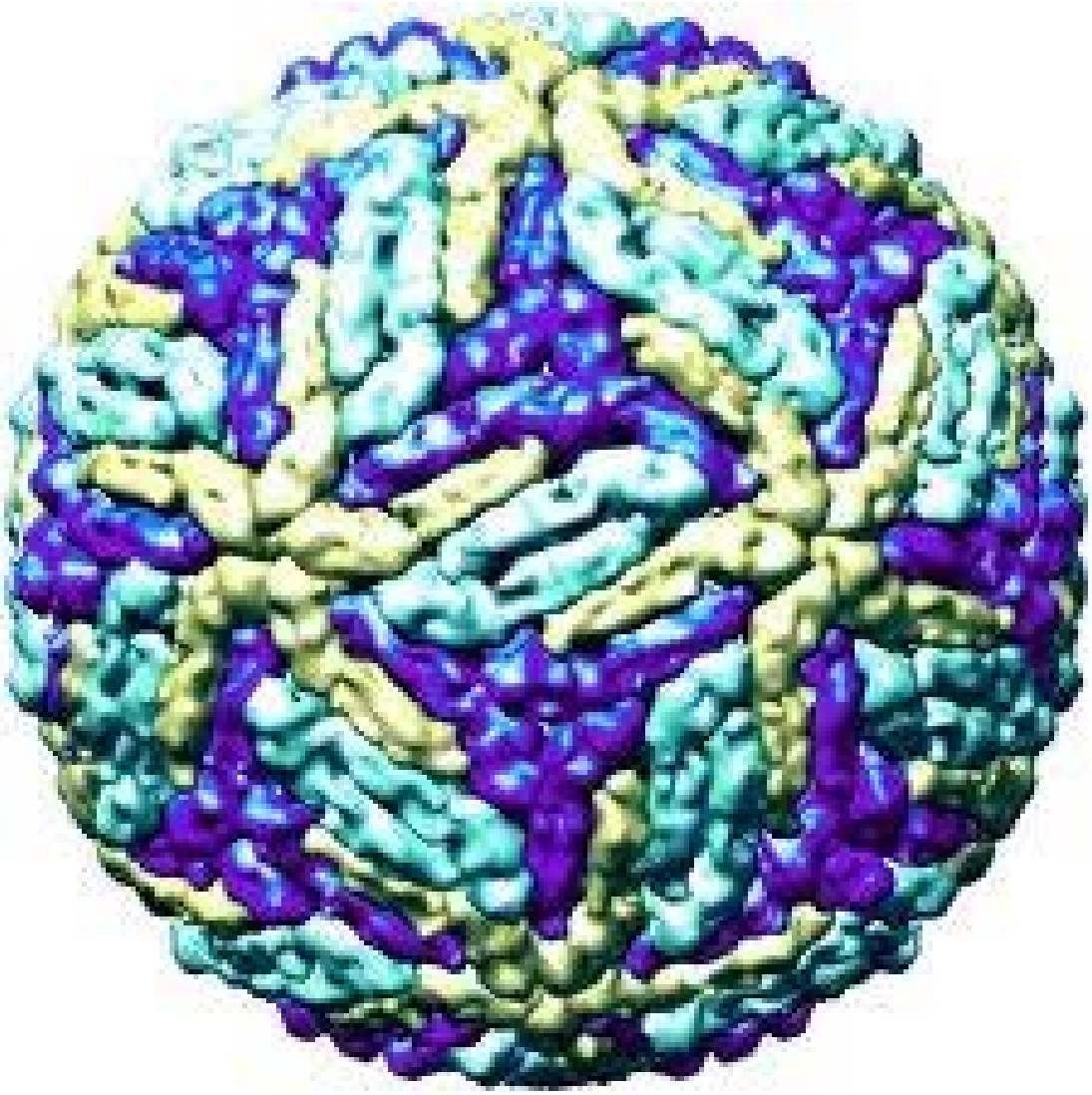
$$Q'(t)= rQ(t)$$ Ecuación diferencial

Modelo de compartimentos

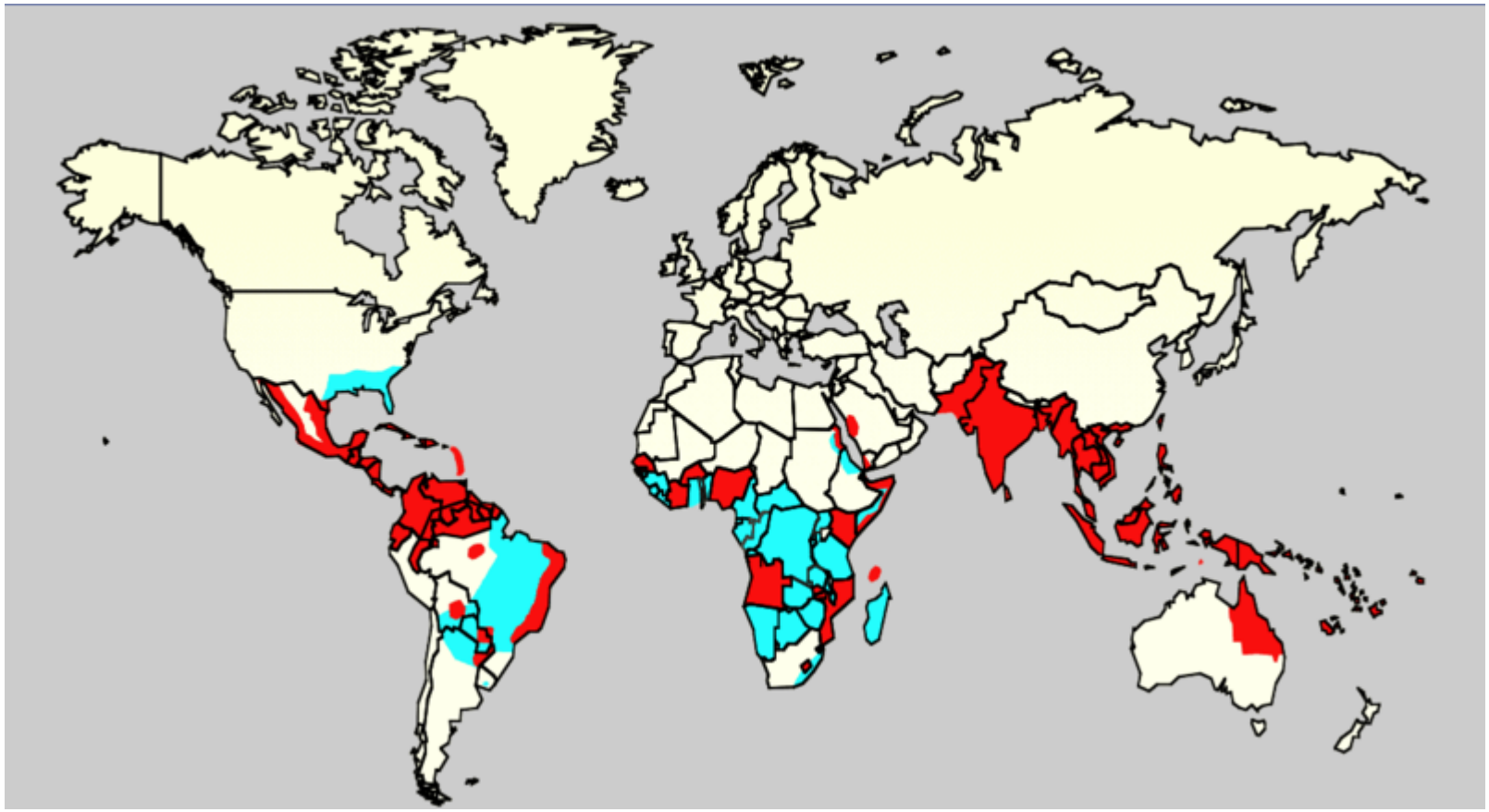


Aedes aegypti

Virus del Dengue



Distribución de Dengue mundial (2006)



Cinche de agua

Trypanosoma cruzi

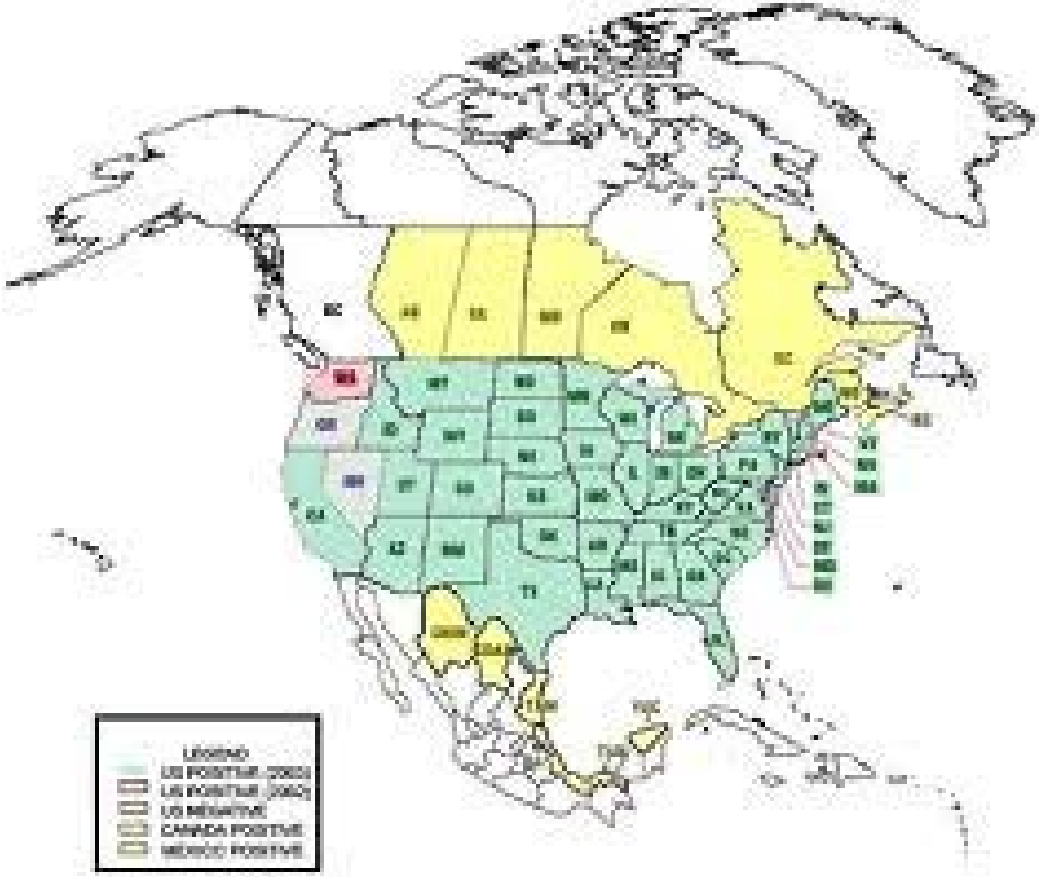
Niño enfermo de Chagas

Corazón carcomido

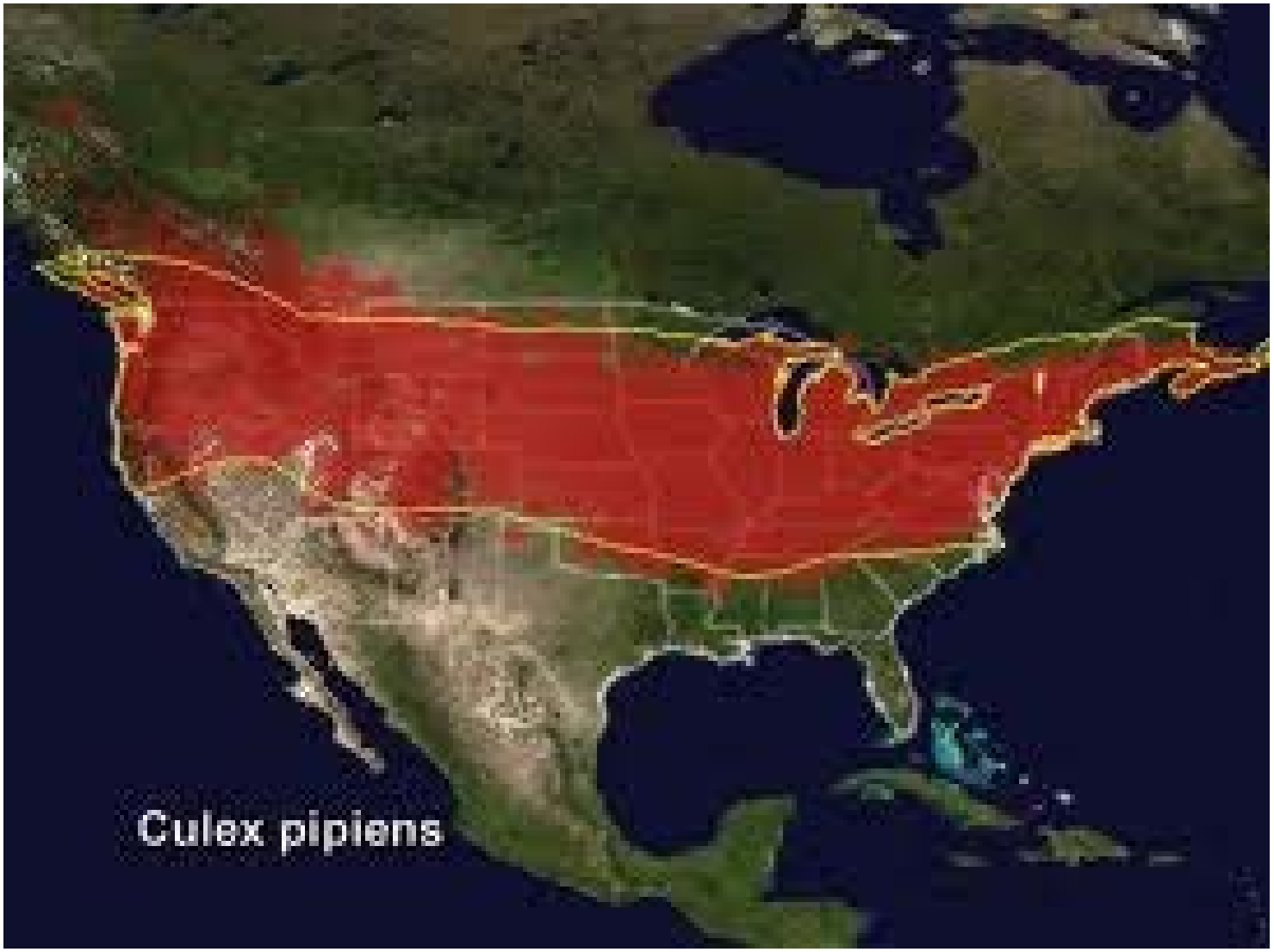
Culex asociado a WNV

Culex quinquefasciatus

Distribución de WNV en América



Distribución del Culex pipiens



Artículos fundamentales

1. W. O. Kermack and A. G. McKendrick, ***A Contributions to the Matematical Theory of Epidemics***. Proc. Roy. Soc. A. Vol. 141, pp. 94-122 (1933).

2. Vito Volterra, ***Variations and Fluctuations of Population Size in Coexisting Animal Species***. Acad. Lincei. 2, pp. 31-113 (1926).

3. Lotka, A. J., ***Elements of Physical Biology***, Williams & Wilkins, Baltimore. (1925). Reprinted under the title Elements of Mathematical Biology. Dover, N. Y. (1956) .

4. Ross, R. ***The Prevention of Malaria***. Murray, London (1911).

5. Hodgkin, A. L. and Huxley, A. F., ***A Quantitative Description of Membrane Current and its Application to Conduction and Exitation in Ne***rve. J. Physiol (London) 117, pp. 100-544 (1952)

6. Kolmogorov, A., Petrovsky, I. and Piskounov, N***. Etude del L'équation de la diffusion avec croissance de la quantité de matière et son application à un problème biologique,*** Bull. Universitè d'Etat à Moscow. (1937)

Artículos fundamentales

7. Turing , A. M. ***The Chemical Basis of Morphogenesis***. Phil. Trans. Roy. Soc. Lond. B237, pp. 37-72. (1952)

8. Lanchester, F. W. ***Aircraft in Warfare: The Dawn of the Fourt Arm, Constable and Colin***.

Modelling Chagas Disease

Gustavo Cruz Pacheco[1], Lourdes Esteva[2], Cristobal Vargas[3]
[1]IIMAS-UNAM, [2]Facultad de Ciencias-UNAM, [3] CINVESTAV-IPN

Abstract

- Formulation of a model for the dynamics of Chagas disease considering vectors, humans, transmitters (mammals) and non-transmitters (birds).
- Main objective: to assess the effectiveness of disease control measures.
- Sensitivity analysis of the basic reproductive number R_0 with respect to epidemiological and demographic parameters.

Introduction

- Chagas disease (American trypanosomiasis) is a life-threatening illness caused by the protozoan parasite, Trypanosoma cruzi (T. cruzi).
- Main mode of transmission: Bites of triatomine bug (kissing bugs).
- Also spread through blood transfusion, organ transplantation, and from mother to fetus (1-10 %).
- All species of Triatominae are capable to transmit Chagas disease.
- Most important: Triatoma infestans, Rhodnius prolixus, Triatoma dimidiata, Triatoma brasiliensis, and Panstrongylus megistus.

Kissing bug

T. cruzi

- T. cruzi has been found in more than 100 mammalian species in Latin America and south of U.S.
- Dogs and cats are important link in the transmission to humans.
- Birds and reptiles are resistant to T. cruzi infection.
- Birds may be important sources of blood meals for triatomines

Clinical manifestations

- Early acute phase:
  - Mild symptoms (local swelling at the site of infection)
  - Antiparasitic treatment with benznidazole and nifurtimox with 60-90 % cure rates.
  - Length: 4-8 weeks.
- Chronic phase:
  - 60-80 % of chronically infected are asymptomatic through their lifetime.
  - 20-40 % of chronically infected eventually develop life-threatening heart and digestive system disorders.
  - Antiparasitic treatments delay or prevent the development of disease symptoms.
  - Duration: 10-20 years.

Early symptoms

Rotten heart

- Chagas disease is endemic in poor, rural areas of Latin America.
- Argentina, Brazil, Chile, Paraguay and Uruguay signed the *Cono Sur Iniciative* in 1991.
- Simultaneous campaign to stop the transmission of Chagas disease:
  - Eliminating domestic T. infestant, and other species,
  - Screening of blood donors.
- T. infestans has been eliminated from most of Chile, Brazil and Uruguay, and some regions of Argentina and Paraguay.
- Despite campaigns as *Cono Sur Iniciative*, Chagas disease is consider the most important vector borne infection in Latin America.
- 16-18 millions of persons infected with T. cruzi, 20,000 deaths each year, and 100 millions at risk.

Control measures

- Control measures are limited:
  - no vaccines are available.
  - drugs are effective only in the acute and early chronic phase of infection, and have adverse effects.
  - Control measures:
    - elimination of the vector,
    - screening blood donors,
    - treatment to patients in the acute phase,
    - zooprophylaxis: to attract vectors to domestic animals in which the pathogen cannot amplify (a dead-end host). Rcommended by WHO since 1982.

- Chagas disease is most common among people who live in substandard housing in rural and semi rural areas.
- Most cases are acquired by exposure to insects in domestic or peridomestic cycles, or by congenital transmission.
- We focus on the transmission of Chagas infection taking into account the domestic structure in rural villages, where animals and humans are in continuous contact.

Mathematical models

- Effect of demographic factors and blood transmission. Busenberg, S. N. and Vargas, C. in O. Arino, D. E. Axelrod and M. Kimmel, eds. Dekker, New York 1991 pp. 283–295.
- Vector dynamics and blood transfusion. Velasco-Hernández, J.X (1994). Theor. Pop. Biol. 46 (1): 1-31.
- Age-dependent infectivity. Inaba, H. and Sekine, H. (2004) Math. Biosc. 190 (1): 39-69m.
- Role of domestic animals in Chagas transmissios. Cohen J.E. and Gürtler R. E. (2001). Science 293: 694-698.

Variables

- Humans.
  $N_h = S_h + I_a + I_c$
  - S_h susceptibles, I_a acute infectious, I_c chronic infectious.
- Transmitters. Mammals that transmit the infection.
  $N_t = S_t + I_t$
  - S_t susceptibles, I_t infectious.
- Non-transmitters. Animals that do not transmit the infection.
  N_{nt}
- Vectors. Triatomines.
  $N_v = S_v + I_v$
  - S_v susceptibles, I_v infectious.

Populations Dynamics

- Non-transmitter population $\bar{N}_{nt}$.
- Human and transmitter populations:
  - Constant recruitment rates Λ_h, Λ_t
  - Mortality rates μ_h, μ_t
- Vector population:
  - Recruitment rate: $\phi_h b_h N_h + \phi_t b_t N_t + \phi_{nt} b_{nt} \bar{N}_{nt}$
    - b_h, b_t, b_{nt} number of triatomine bites per day in humans, transmitters, and non-transmitters.
    - ϕ_h, ϕ_t, ϕ_{nt} egg-production rates due to blood meals from human, transmitter, and non-transmitters.
  - Mortality rate μ_v.

Epidemiological Parameters

- Infection rates per susceptible human and transmitter:
$$\frac{b_h \beta_h}{N} I_v, \quad \frac{b_t \beta_t}{N} I_v$$
β_h, β_t transmission probabilities from vector susceptible humans and transmitters.
- Infection rate per susceptible vector:
$$\frac{b_h \alpha_a I_a + b_h \alpha_c I_c + b_t \alpha_t I_t}{N}$$
α_a, α_c, α_t transmission probabilities from infective humans and transmitters to susceptible vectors.

- p proportion of newborns that are acute infected due to vertical transmission.
- γ proportion of acute infectious that become chronic infectious.
- q proportion of cured acute infectious that return to the susceptible class.
- σ disease-induced death rate in the chronic stage.

The model

$$
\begin{aligned}
\frac{dI_a}{dt} &= p\frac{\Lambda_h}{N_h}I_c + \frac{b_h\beta_h}{N}(N_h - I_a - I_c)I_v - (\gamma + \mu_h)I_a \\
\frac{dI_c}{dt} &= (1-q)\gamma I_a - (\mu_h + \sigma)I_c \\
\frac{dI_t}{dt} &= \frac{b_t\beta_t}{N}(N_t - I_t)I_v - \mu_t I_t \\
\frac{dI_v}{dt} &= \frac{b_h\alpha_a I_a + b_h\alpha_c I_c + b_t\alpha_t I_t}{N}(N_v - I_v) - \mu_v I_v
\end{aligned}
$$

$$S_h = N_h - I_a - I_c,\ S_t = N_t - I_t,\ S_v = N_v - I_v$$

The population size of the three species satisfy the equations:

$$\begin{aligned}
\frac{dN_h}{dt} &= \Lambda_h - \sigma I_c - \mu_h N_h \\
\frac{dN_t}{dt} &= \Lambda_t - \mu_t N_t \\
\frac{dN_v}{dt} &= \phi_h b_h N_h + \phi_t b_t N_t + \phi_{nt} b_{nt} \bar{N}_{nt} - \mu_v N_v .
\end{aligned}$$

Basic reproductive number R_0

- In the absence of disease:

$$\bar{N}_h = \frac{\Lambda_h}{\mu_h} \quad \bar{N}_t = \frac{\Lambda_t}{\mu_t} \quad \bar{N}_v = \frac{\phi_h b_h \bar{N}_h + \phi_t b_t \bar{N}_t + \phi_{nt} b_{nt} \bar{N}_{nt}}{\mu_v}$$

$$\boxed{R_0 = \sqrt{\frac{b_h^2 \beta_h H \bar{N}_h \bar{N}_v}{\mu_v K \bar{N}^2} + \frac{b_t^2 \beta_t \alpha_t \bar{N}_t \bar{N}_v}{\mu_v \mu_t \bar{N}^2}}}$$

$$H = [(\mu_h + \sigma)\alpha_a + (1-q)\gamma\alpha_c]$$

$$K = (\gamma_h + \mu_h)(\sigma + \mu_h) - p(1-q)\gamma\mu_h$$

$$R_0 = \sqrt{R_h^2 \left(\frac{\bar{N}_h + \bar{N}_{nt}}{\bar{N}} \right)^2 + R_t^2 \left(\frac{\bar{N}_t + \bar{N}_{nt}}{\bar{N}} \right)^2}$$

$$R_h = \sqrt{\frac{b_h^2 \beta_h H \bar{N}_v \bar{N}_h}{\mu_v K (\bar{N}_h + \bar{N}_{nt})^2}}$$

secondary infections in human-vector cycle

$$R_t = \sqrt{\frac{b_t^2 \beta_t \alpha_t \bar{N}_v \bar{N}_t}{\mu_v \mu_t (\bar{N}_t + \bar{N}_{nt})^2}}$$

secondary infections in transmitter-vector cycle

Results ($\sigma = 0$)

- Disease free equilibrium $P_0 = (0, 0, 0, 0)$
- P_0 is g.a.s. if $R_0 \leq 1$, unstable if $R_0 > 1$
- Lyapunov function

$$\mathcal{V} = b_1 I_a + b_2 I_c + b_3 I_t + I_v$$

$$\begin{aligned} b_1 &= \frac{b_h \alpha_a \bar{N}_v}{K\bar{N}} + \frac{b_h \alpha_c (1-q)\gamma \bar{N}_v}{\mu_h K \bar{N}} \\ b_2 &= \frac{p b_h \alpha_a \bar{N}_v}{K\bar{N}} + \frac{(\gamma + \mu_h) b_h \alpha_c \bar{N}_v}{\mu_h K \bar{N}} \\ b_3 &= \frac{b_t \alpha_t \bar{N}_v}{\mu_t \bar{N}} \end{aligned}$$

Endemic equilibrium

- $P_1 = (\bar{I}_a, \bar{I}_c, \bar{I}_t, \bar{I}_v)$

$$\bar{I}_a = \frac{\mu_h b_h \beta_h \bar{N}_h \bar{I}_v}{\bar{N} K \mu_h + K_1 \bar{I}_v} \quad \bar{I}_c = \frac{(1-q)\gamma}{\mu_h} \bar{I}_a \quad \bar{I}_t = \frac{b_t \beta_t \bar{N}_t \bar{I}_v}{b_t \beta_t \bar{I}_v + \mu_t \bar{N}}$$

$$p(\bar{I}_v) = a_1 \bar{I}_v^2 + a_2 \bar{I}_v + a_3$$

$$K_1 = b_h \beta_h (\mu_h + (1-q)\gamma)$$

- P_1 is feasible if $R_0 > 1$
- Linear analysis: P_1 is l.a.s. if $R_0 > 1$

- $q = 0$, $p = 0$. Lyapunov function:

$$\begin{aligned}
\mathcal{U} &= c_1(S_h - \bar{S}_h - \bar{S}_h \ln \frac{S_h}{\bar{S}_h}) + c_2(I_a - \bar{I}_a - \bar{I}_a \ln \frac{I_a}{\bar{I}_a}) \\
&+ c_3(I_c - \bar{I}_c - \bar{I}_c \ln \frac{I_c}{\bar{I}_c}) \\
&+ c_4(S_t - \bar{S}_t - \bar{S}_t \ln \frac{S_t}{\bar{S}_t}) + c_5(I_t - \bar{I}_t - \bar{I}_t \ln \frac{I_t}{\bar{I}_t}) \\
&+ c_6(S_v - \bar{S}_v - \bar{S}_v \ln \frac{S_v}{\bar{S}_v}) + c_7(I_v - \bar{I}_v - \bar{I}_v \ln \frac{I_v}{\bar{I}_v}),
\end{aligned}$$

$$\bar{S}_h = N_h - \bar{I}_a - \bar{I}_c, \quad \bar{S}_t = N_t - \bar{I}_t, \quad \bar{S}_v = N_v - \bar{I}_v$$

- P_1 is g.a.s.

Control measures. Reducing vector population

- Reducing vector population by insecticides (or other means) $\Rightarrow$ vector mortality μ_v increases.
- Increasing μ_v by a factor $\theta > 1 \Rightarrow R_0$ decreases by a factor $1/\theta$.
- Insecticide application every 3 months $\Rightarrow$ life span $/\mu_v$ decreases from 6 months to 3 months $\Rightarrow \mu_v$ increases twice, and R_0 decreases by half.
- The disease could be controlled by mere application of insecticide every three months, if $\dfrac{R_0}{2} < 1$.

Control measures. Screening and treatment

- Early treatment and cure of acute infected $\Rightarrow$ q increases by a factor $\theta > 1$, $\theta q \leq 1$.

$$\begin{aligned} R_0(\bar{q}) &\approx R_0(q) + \frac{\partial R_0}{\partial q}(1-\theta)q \\ &= R_0(q)\left[1 - \frac{Q}{2R_0^2(q)}\right](\theta - 1)q \end{aligned}$$

$$Q = R_h^2 \frac{(N_h + N_{nt})^2}{N^2} \frac{q\gamma(p\alpha_a\mu_h + \alpha_c(\mu_h + \gamma))}{K(\mu_h\alpha_a + (1-q)\gamma\alpha_c)}$$

- $\theta = 1/q$. Minimum value $R_0\left[1 - \frac{Q}{2R_0^2(q)}\right](1-q)$

Control measures. Zooprophylaxis

- Population of non- transmitters N_{nt} increases to $\bar{N}_{nt} = \theta N_{nt}$, $\theta > 1$.

$$R_0(\bar{N}_{nt}) \approx R_0(N_{nt}) + \frac{\partial R_0(N_{nt})}{\partial N_{nt}}(\theta - 1)N_{nt}$$

$$\frac{\partial R_0}{\partial N_{nt}} = R_0 \frac{[(\phi_{nt} b_{nt} - 2\phi_h b_h)N_h + (\phi_{nt} b_{nt} - 2\phi_t b_t)N_t - \phi_{nt} b_{nt} N_{nt}]}{2N\bar{N}_v}$$

- $R_0(N_{nt})$ has a unique local maximum positive when

$$(\phi_{nt} b_{nt} - 2\phi_h b_h)N_h + (\phi_{nt} b_{nt} - 2\phi_t b_t)N_t > 0$$

- Depending on the density, biting rate, and egg-production rate from blood meals of the populations involved, R_0 increases or decreases.

parameter	value	sources
p	$2\,\% - 10\,\%$	Kirchhoff, 2010
b_h	0.04 day^{-1}	Cohen, 2001
b_t	0.1 day^{-1}	Cohen, 2001
b_{nt}	0.12 day^{-1}	Cohen, 2001
β_h	0.0009	Rabinovich, 1999
β_t	0.0009	Rabinovich, 1999
α_a	0.03	Cohen, 2001
α_c	0.03	Cohen, 2001
α_t	0.49	Gurtler, 1996
$1/\gamma$	28-56 days	Bern, 2007
q	0.04 %	
$1\mu_h$	65 years	
$1/\mu_t$	10 years	
$1/\mu_{nt}$	7 years	
$1/\mu_v$	7 months	Rabinovich, 1972

Table 1. Parameters in the numerical simulations of Chagas model.

$\bar{N}_h$	human population	5000
$\bar{N}_t$	transmitter population	$2/5 \times N_h$
$\bar{N}_{nt}$	non-transmitter population	$3/5 \times N_h$
$\bar{N}_v$	vector population	$100 \times N_h$

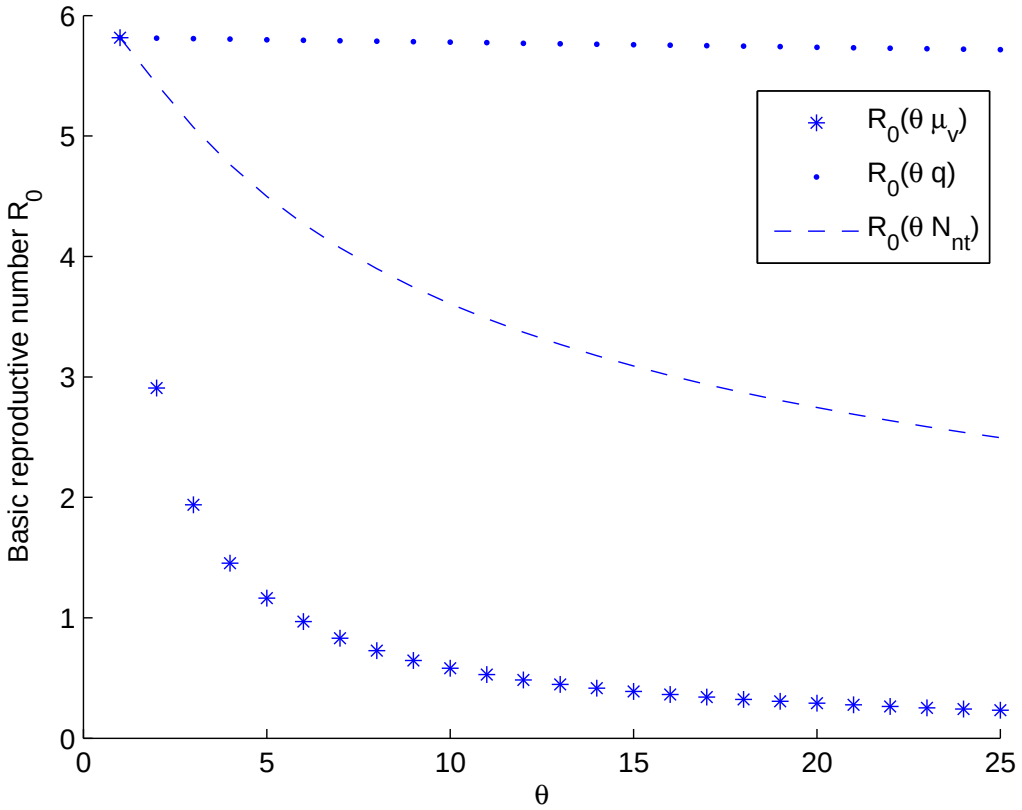


Figure 1. Evolution of R_0 as a function of the control variables $\theta\mu_v$, θq, and θN_{nt}, $1 \leq \theta \leq 5$. Initial $R_0 = 5{,}8$

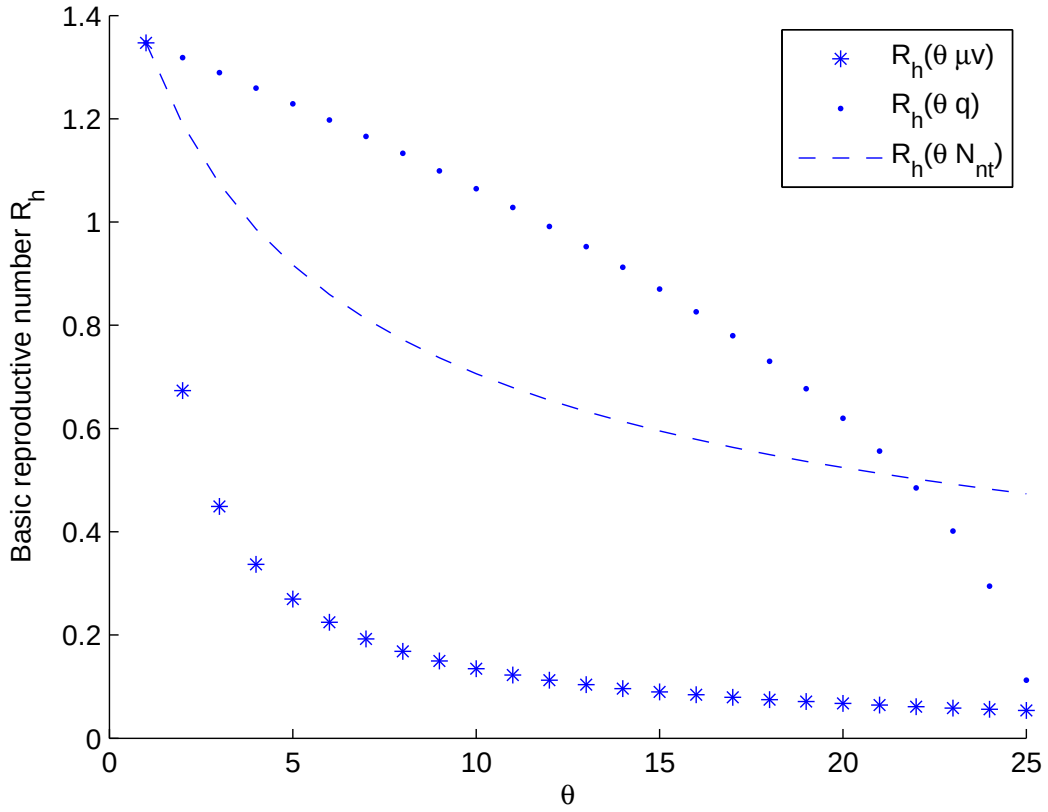


Figure 2. Evolution of R_h as a function of the control variables $\theta\mu_v$, θq, and θN_{nt}, $1 \leq \theta \leq 5$. Initial $R_0 = 1{,}3$

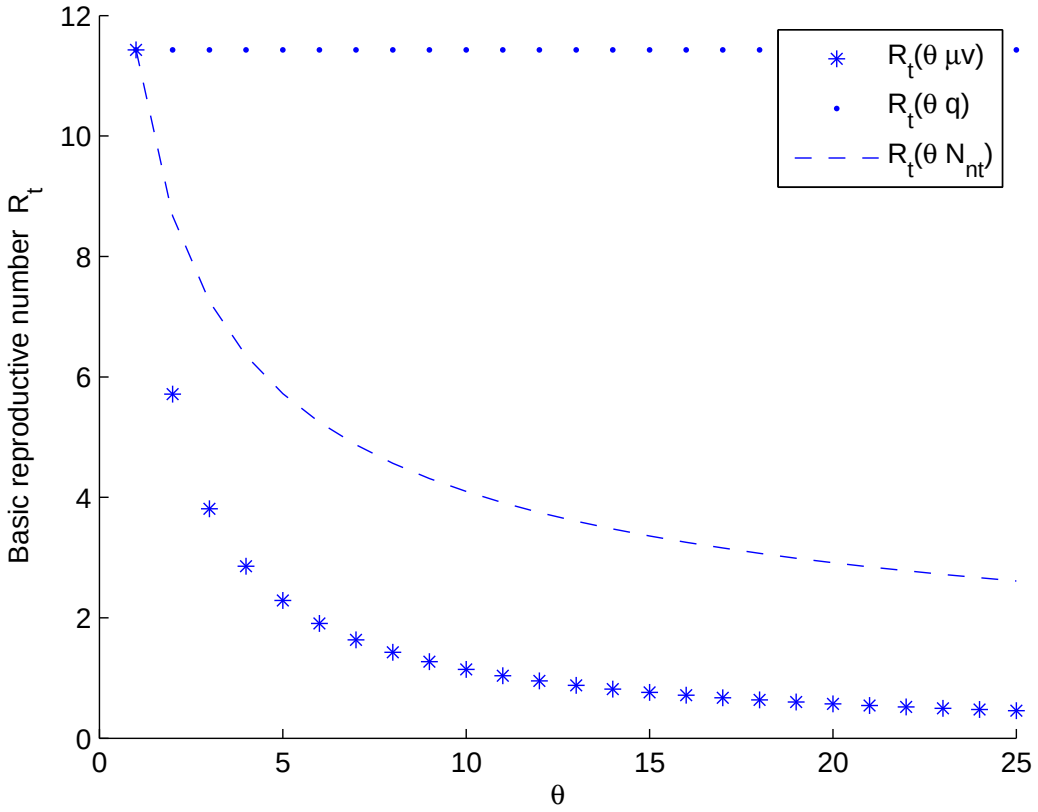


Figure 3. Evolution of R_t as a function of the control variables $\theta\mu_v$, θq, and θN_{nt}, $1 \leq \theta \leq 5$. Initial $R_0 = 11{,}4$

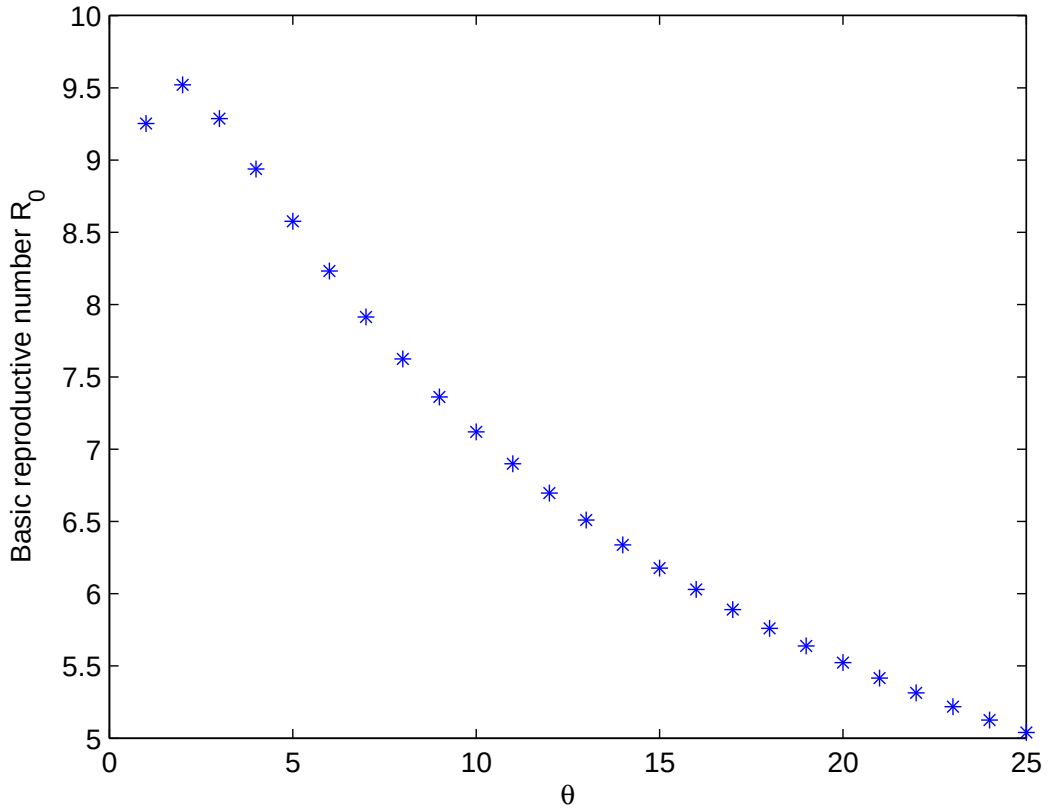


Figure 4. Evolution of R_0 as a function of θNnt with $b_{nt} = 0{,}5$ and other parameters as in Table 1.

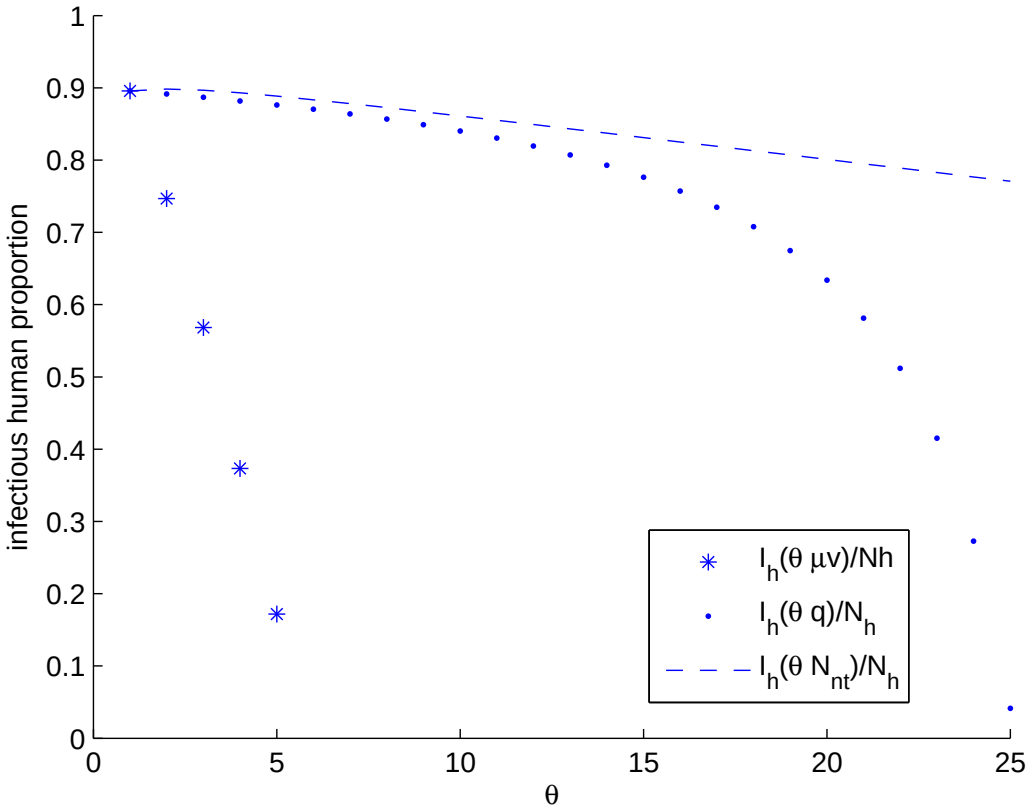


Figure 5.Endemic human proportions I_h/N_h with $I_h = I_a + I_c$ as a function of the control variables $\theta\mu_v$, θq, and θN_{nt}.

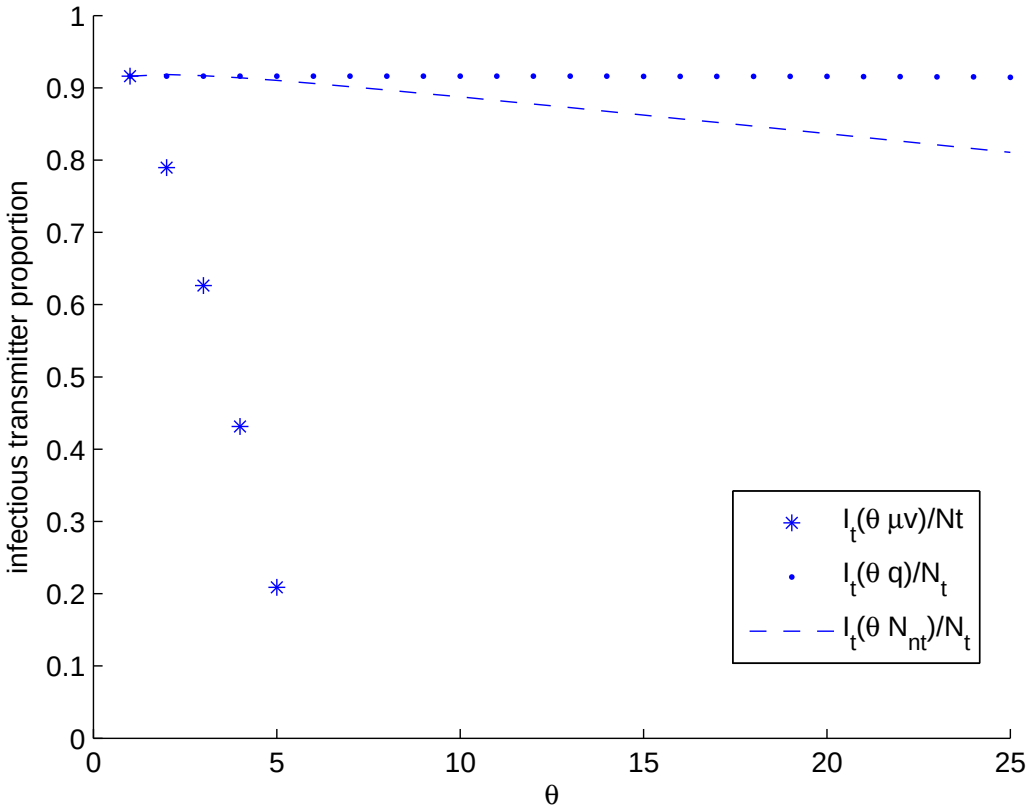


Figure 6.Endemic transmitter proportions I_t/N_t as a function of the control variables $\theta\mu_v$, θq, and θN_{nt}.

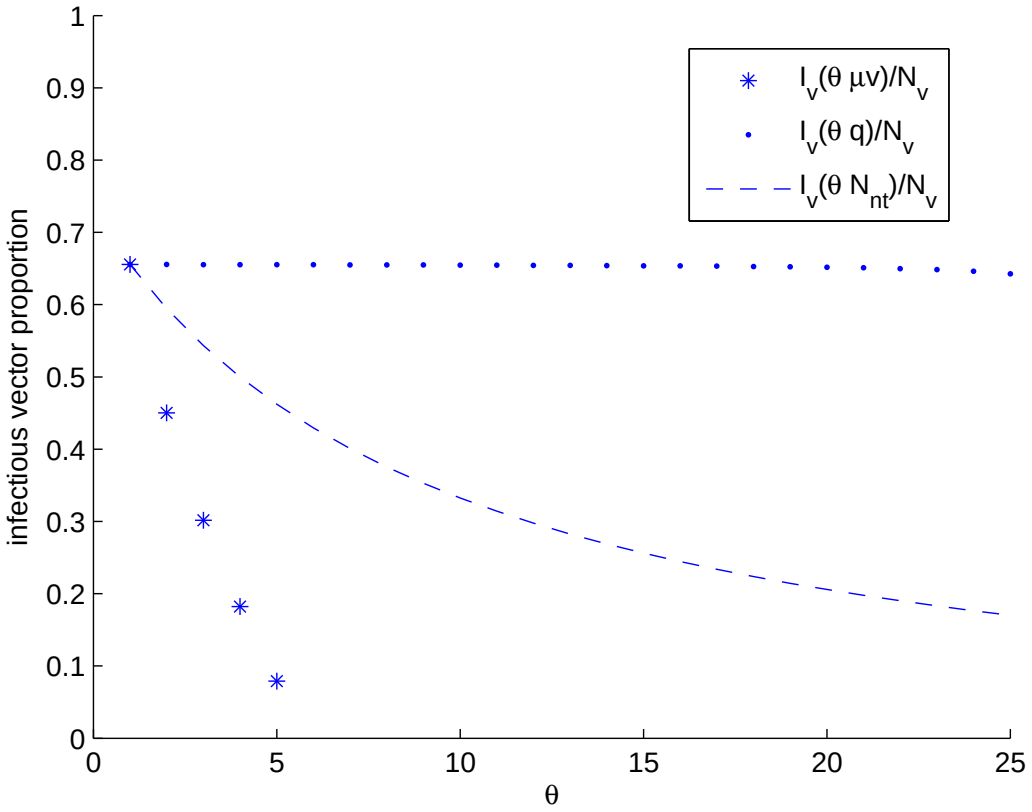


Figure 7. Endemic vector proportions I_v/N_v as a function of the control variables $\theta\mu_v$, θq, and θN_{nt}.

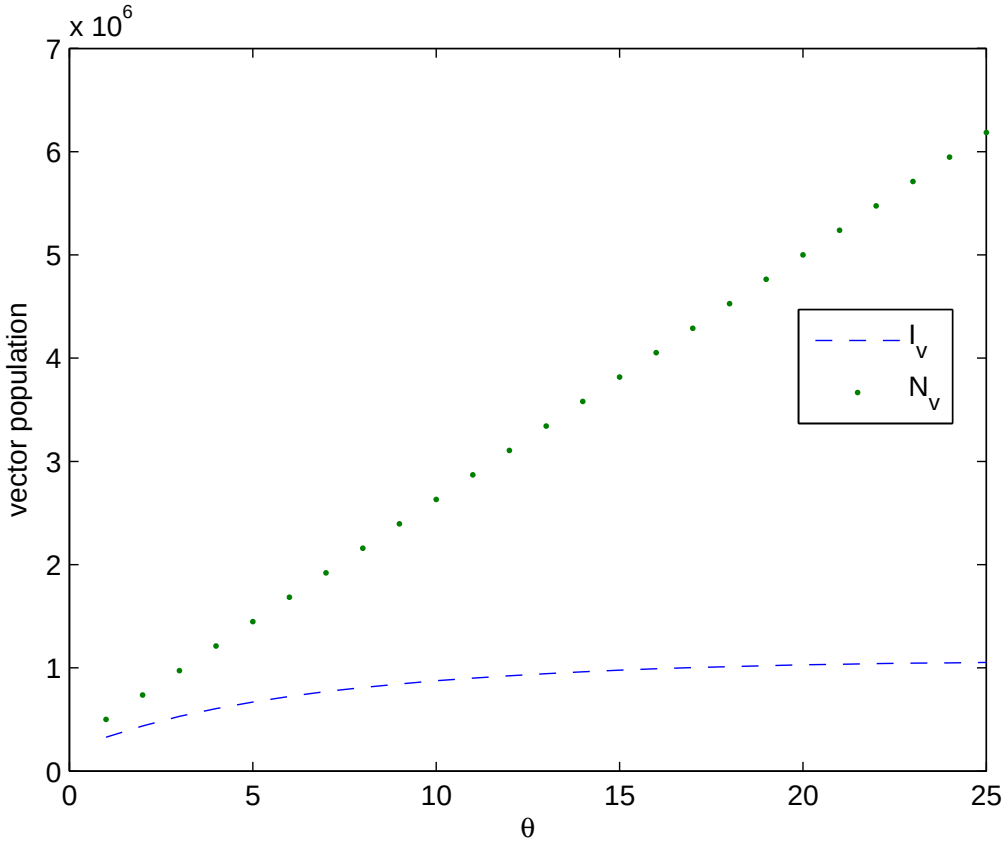


Figure 8. Population size of triatomines and infected triatomines as a function of the number of non-transmiters N_{nt}.

Relative rate of decrease

$$\eta = \frac{p_i - p_f}{p_i}$$

p_i and p_f human infectious at the start and end of the control.

θ	2	3	4	5
Increment of mortality μ_v	0.16	0.37	0.58	0.80
Increment of treatment q	0.005	0.009	0.01	0.02
Increment of non-transmitters N_{nt}	-0.003	$-0{,}0008$	0.003	0.008

- Elimination of vectors is by far the best way to control the endemic levels, followed by early detection and treatment of acute cases.
- Zooprophylaxis has no major impact in reducing the disease.

Conclusions

- The model corroborates that reduction of vectors is the most effective measure to control the disease.
- Early detection and treatment of disease is very important, however, the model shows that close proximity to domestic animals may prevent this treatment as a good control measure.
- Increasing the number of non-transmitter animals at home, decreases very slightly the proportions of infected humans and animals. Under certain conditions, ths measures increases the number of new infections.
- Reducing the population of triatomines, and keeping domestic animals out of the dormitories in order to reduce animal transmission seems to be best way to decrease the risk of human infections as mention in (Cohen, 2001).

Multispecies Interactions in West Nile Virus

GUSTAVO CRUZ-PACHECO
IIMAS, UNAM

LOURDES ESTEVA
Facultad de Ciencias,UNAM

CRISTÓBAL VARGAS
Departamento de Control Automático, CINVESTAV-IPN

Introduction

- West Nile Virus (WNV) is an arbovirus transmitted by mosquito's bites to different species of birds and mammals, including humans.
- Transmission cycle is between mosquitoes and birds. Mammals can not transmit the virus.
- Mosquitoes present vertical transmission.

- WNV was detected the first time in the american continent in 1999 in New York City. (www.cdc.gov/ncidod/dvbid/westnile/).
- Since then, it has spread to almost all of the United States and parts of Canada.
- The virus have been found in serological tests in birds and horses in Latin America and Carribean islands.
- Low incidence of WNV in these regions.
- Possible explanations:
  - Cross immunity with other flavivirus like dengue.
  - Mutations of the virus in its way to south (Hayes, 2005)

Important questions

- The role of different species of birds and mosquitoes in WNV outbreaks.
- The effect of the interaction among the bird and mosquito species in the emergency and prevalence of WNV.

[Turell, M.J. *et al.* (2005). An update on the potential of north American mosquitoes (Diptera: Culicidae) to transmit West Nile virus. J. Med Entomol. 42: 57-62.]

[Komar, N. (2003). West Nile virus: epidemiology and ecology in North America. Adv. Virus Res. 61:185-234.]

The basic model

- N_a, N_v bird and mosquito populations.
- $S_a(t)$, $S_v(t)$ susceptible birds and mosquitoes.
- $R_a(t)$ recovered birds.
- μ_v mortality rate of mosquitoes.
- μ_a mortality rate of birds.
- Λ_a recruitment rate of bird population.
- b biting rate.
- β_a, β_v transmission probabilities.
- γ_a birds recovered rate.
- α_a disease mortality rate.

$$
\begin{aligned}
\frac{dS_a}{dt} &= \Lambda_a - \frac{b\beta_a}{N_a} I_v S_a - \mu_a S_a \\
\frac{dI_a}{dt} &= \frac{b\beta_a}{N_a} I_v S_a - (\gamma_a + \mu_a + \alpha_a) I_a \\
\frac{dR_a}{dt} &= \gamma_a I_a - \mu_a R_a \\
\frac{dS_v}{dt} &= \mu_v S_v - \frac{b\beta_v}{N_a} I_a S_v - \mu_v S_v \\
\frac{dI_v}{dt} &= \frac{b\beta_v}{N_a} I_a S_v - \mu_v I_v \\
\frac{dN_a}{dt} &= \Lambda_a - \mu_a N_a - \alpha_a I_a
\end{aligned}
$$

$$S_a + I_a + R_a = N_a, \qquad S_v + I_v = N_v$$

.

Basic Reproductive Number R_0

$$R_0 = \sqrt{\frac{mb\beta_v}{\gamma_a + \mu_a + \alpha_a} \cdot \frac{b\beta_a}{\mu_v}}$$

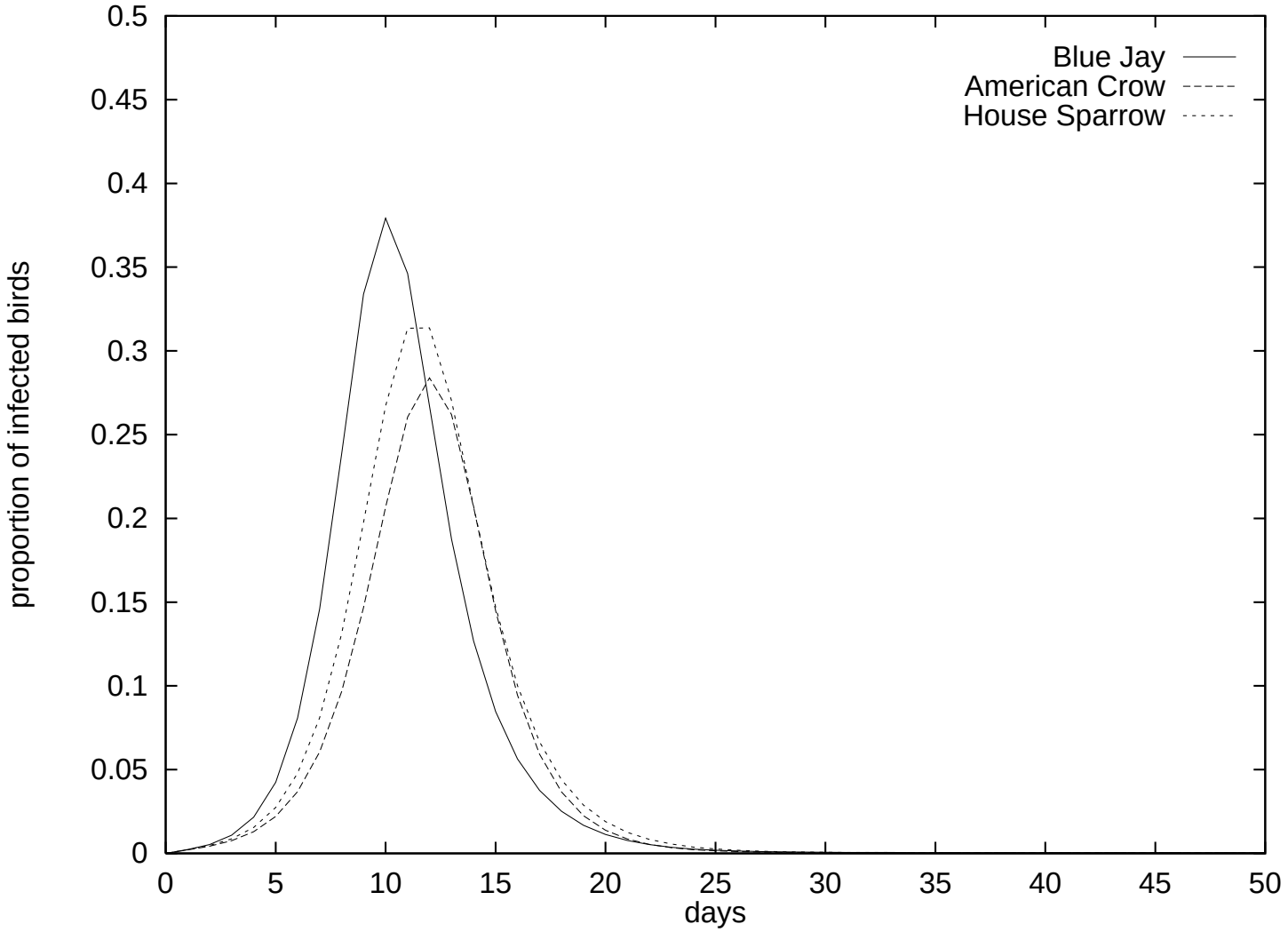
$$m = \frac{N_v}{N_a}$$

Results

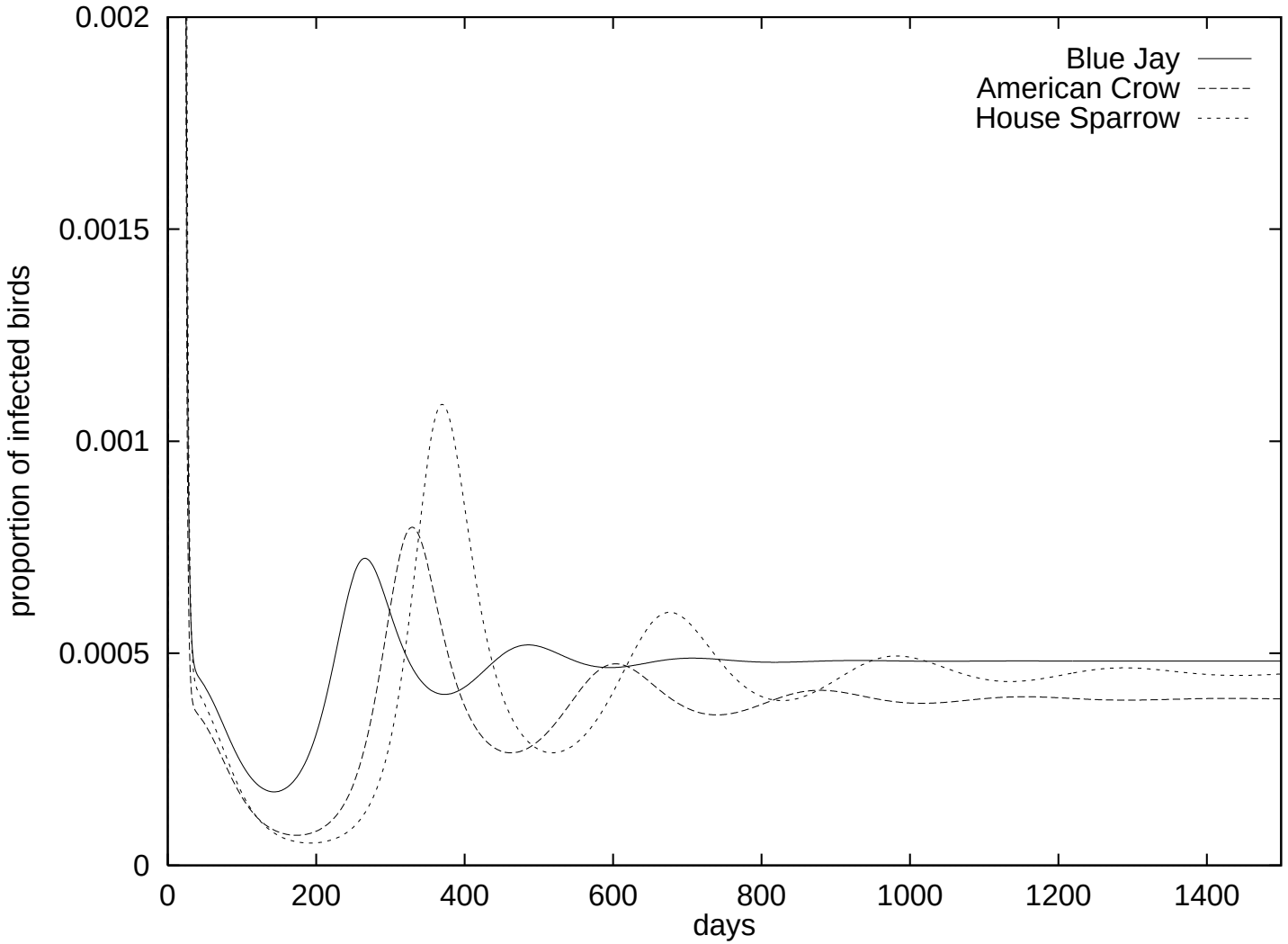
- If $R_0 < 1 \Rightarrow$ disease-free equilibrium E_0 g.a.s.
- If $R_0 > 1 \Rightarrow$ endemic equilibrium E_1 l.a.s.

name	β_a	β_v	γ_a	α_a	μ_a	μ_v	m	R_0
Blue Jay	1.0	0.68	0.26	0.15	.0002	.06	5	5.89
Common Grackle	1.0	0.68	0.33	.07	.0001	.06	5	6.97
House Finch	1.0	0.32	0.18	0.14	.0003	.06	5	4.57
American Crow	1.0	0.5	0.31	0.19	.0002	.06	5	4.58
House Sparrow	1.0	0.53	0.33	0.1	.0002	.06	5	5.08
Ring-billed Gull	1.0	0.28	0.22	0.1	.0003	.06	5	4.28
Black-billed Magpie	1.0	0.36	0.33	0.16	.0001	.06	5	3.92
Fish Crow	1.0	0.26	0.36	.06	.0002	.06	5	3.60

Epidemiological and demographic parameters. Units days^{-1}.



Temporal course of infected birds. $s_a = 1$, $i_a = 0$, $i_v = 0.001$, $n_a = 1$.



Asymptotic behavior of solutions.

N species of birds

- N_i Total population of i bird species
- S_i Susceptible population of i bird species
- I_i Infectious population of i bird species

$$
\begin{aligned}
\frac{dS_i}{dt} &= \Lambda_i - \frac{b\beta_i}{N_1 + ... + N_n} S_i I_v - \mu_i S_i \\
\frac{dI_i}{dt} &= \frac{b\beta_i}{N_1 + ... + N_n} S_i I_v - (\gamma_i + \mu_i + \alpha_i) I_i \\
\frac{dI_v}{dt} &= \sum_{i=1}^{n} \frac{b\beta_{v_i} I_i}{N_1 + ... + N_n} (N_v - I_v) - \mu_v I_v \\
\frac{dN_i}{dt} &= \Lambda_i - \mu_i N_i - \alpha_i I_i
\end{aligned}
$$

$i = 1, .., n.$

Basic Reproductive Number R_0^a

$$R_0^a = \sqrt{\sum\nolimits_{i=1}^{n} R_i^2 \left(\bar{N}_i/\bar{N}\right)}$$

with

$$R_i = \sqrt{\frac{b^2 \beta_i \beta_{v_i}}{\mu_v(\gamma_i + \mu_i + \alpha_i)} \left(N_v/\bar{N}\right)}$$

the basic reproductive number of the i bird species, $i = 1, ..., n$.

$\bar{N}_i = \Lambda_i/\mu_i$, $\bar{N} = \bar{N}_1 + \bar{N}_2 + ... + \bar{N}_n$

Results

- Si $R_0^a < 1 \Rightarrow$ the disease free equilibrium is g.a.s.
- Assume $\alpha_i = 0$, $i = 1, ..n$. If $R_0^a > 1 \Rightarrow$ exists a unique endemic equilibrium g.a.e.
- R_0^a is the average of the basic reproductive numbers of the bird species weighted by their corresponding population proportion with respect to the total number of birds.
- The interplay between the competence of the birds species to transmit the disease and their corresponding population density will determine the evolution of the disease.

Bird	β_v	γ_a	α_a	μ_a
Common Grackle	0.68	0.33	0.07	0.0001
Northern Flicker	0.06	1.0	0.0	0.0003

Data from

1. Komar, N. (2003). West Nile virus: epidemiology and ecology in North America. Adv. Virus Res. 61:185-234

2. The University of Michigan Museum of Zoology, 2004. Animal Diversity Web. http://www.animaldiversity.ummz.umich.edu.

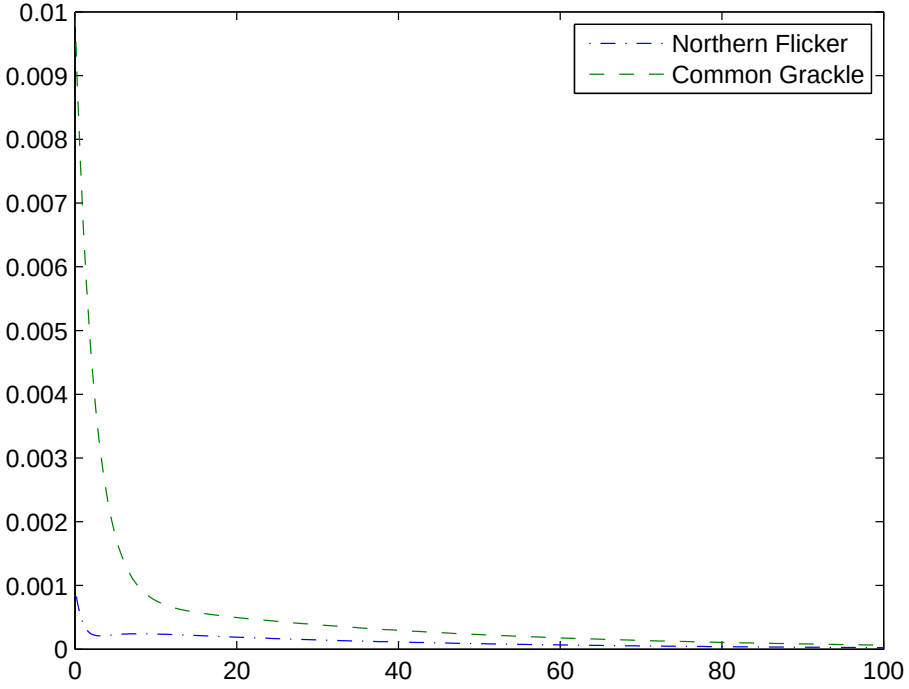
Vector	b	β_a	μ_v
Ae.*albopictus*	0.1	0.86	0.07
Cx.*pipiens*	0.5	0.88	0.07

Data from:

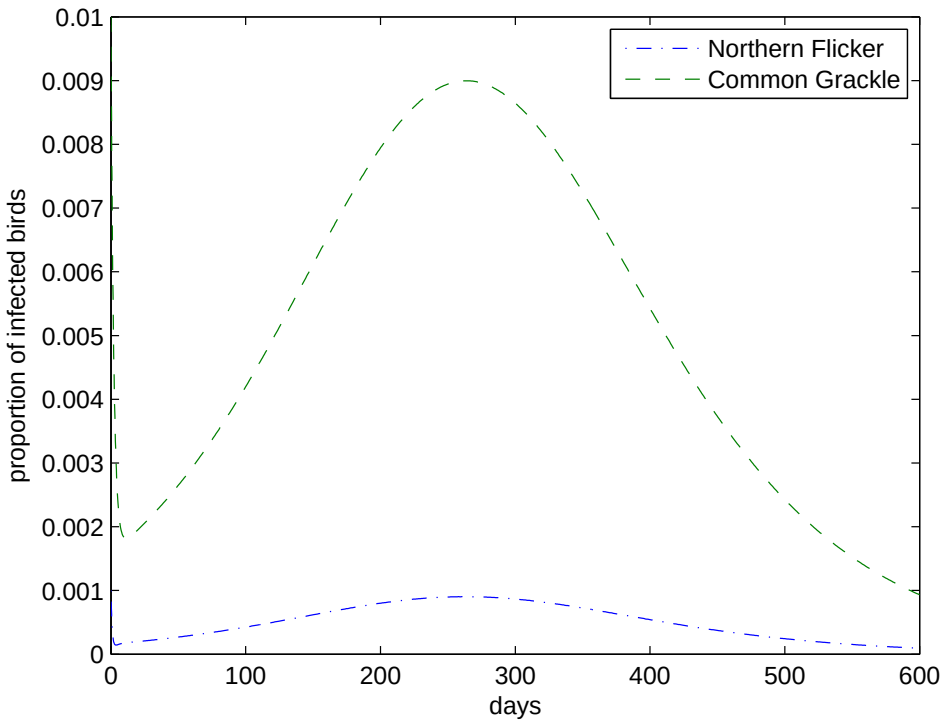
Savage et al. (1993). Host-feeding patterns of *Aedes albopictus* (Diptera:Culicidae) at a temperate North American site. J. Med. Entomol. 30: 27-34.

Turell, et al. (2005). An update on the potential of north American mosquitoes (Diptera: Culicidae) to transmit West Nile virus. J. Med Entomol. 42: 57-62.

Wonham et al. (2004). An epidemiological model for West Nile virus:



Temporal course of the proportion of infected birds. For Northern Flicker $R_{0_1} = 0.41$, and $\bar{N}_1 = 1000$. For Common Grackle, $R_{0_2} = 2.2$, and $\bar{N}_2 = 100$. In this case $R_0 = 0.76$.



Temporal course of the proportion of infected birds. For Northern Flicker, $R_{0_1} = 0.37$, and $\bar{N}_1 = 1000$. For Common Grackle, $R_{0_2} = 1.95$, and $\bar{N}_2 = 400$. In this case $R_0 = 1.2$.

Dependence of R_0^a with respect to parameters.

- R_0^a depends linearly on b, β_i, β_v, N_v.
- $\dfrac{\partial (R_0^a)^2}{\partial N_i} = \dfrac{1}{N}\left(-2(R_0^a)^2 + R_i^2\right)$.
- $\rho_i = R_i^a / R_0^a$:
  - If $\rho_i > \sqrt{2}$, R_0^a increases when N_i increases
  - Si $\rho_i < \sqrt{2}$, R_0^a decreases when N_i increases
- Many bites are wasted in birds poorly efficient to transmit the virus.
- If the population of birds keeps growing, the basic reproduction number decreases.

N species of mosquitoes

- N_{v_i} Total population of i mosquito species
- I_{v_i} Infectious population of i mosquito species

$$
\begin{aligned}
\frac{dS_a}{dt} &= \Lambda_a - \sum_{i=1}^{n} \frac{N_{v_i}}{N_{v_1} + N_{v_2} + \ldots + N_{v_n}} \frac{b_i \beta_{a_i}}{N_a} S_a I_{v_i} - \mu_a S_a \\
\frac{dI_a}{dt} &= \sum_{i=1}^{n} \frac{N_{v_i}}{N_{v_1} + N_{v_2} + \ldots + N_{v_n}} \frac{b_i \beta_{a_i}}{N_a} S_a I_{v_i} - (\gamma_a + \mu_a + \alpha_a) I_a \\
\frac{dI_{v_i}}{dt} &= \frac{b_i \beta_{v_i} I_a}{N_a} (N_{v_i} - I_{v_i}) - \mu_{v_i} I_{v_i} \\
\frac{dN_a}{dt} &= \Lambda_a - \mu_a N_a - \alpha_a I_a
\end{aligned}
$$

$i = 1, .., n.$

$$R_0^v = \sqrt{\sum_{i=1}^{n} R_i^2 \frac{N_{v_i}}{N_v}}$$

$$R_{0_i} = \sqrt{\frac{b_i^2 \beta_{a_i} \beta_{v_i}}{\mu_{v_i}(\gamma_a + \mu_a + \alpha_a)} \frac{N_{v_i}}{N_a}}$$

$$N_v = N_{v_1} + N_{v_2} + ... + N_{v_n}$$

- The disease-free equilibrium is g.a.s. if $R_0^v < 1$.
- Assume $\alpha = 0$. If $R_0^v > 1$ there exists a unique endemic equilibrium g.a.s..

$$\frac{\partial (R_0^v)^2}{\partial N_{v_i}} = \frac{1}{N_v}\left(-(R_0^v)^2 + 2R_{0_i}^2\right).$$

- • R_0^v increases if $\sigma = R_{0_i}/R_0^v > \frac{1}{\sqrt{2}}$.
- • R_0^v decreases if $\sigma = R_{0_i}/R_0^v < \frac{1}{\sqrt{2}}$.
- • If $\sigma_i > \frac{1}{\sqrt{2}}$, R_0 will keep increasing when the mosquito population increases, since R_{0_i} becomes bigger.

n species of birds and m species of mosquitoes

$$
\begin{aligned}
\frac{dS_i}{dt} &= \Lambda_i - \sum_{j=1}^{m} \frac{N_{v_j}}{(N_{v_1} + \cdots N_{v_m})} \frac{b_j \beta_{ij}}{(N_1 + \ldots + N_n)} S_i I_{v_j} - \mu_i S_i \\
\frac{dI_i}{dt} &= \sum_{j=1}^{m} \frac{N_{v_j}}{(N_{v_1} + \cdots N_{v_m})} \frac{b_j \beta_{ij}}{(N_1 + \ldots + N_n)} S_i I_{v_j} - (\gamma_i + \mu_i + \alpha_i) I_i \\
\frac{dI_{v_j}}{dt} &= \sum_{i=1}^{n} \frac{b_j \beta_{v_{ji}}}{N_1 + \ldots + N_n} (N_{v_j} - I_{v_j}) I_i - \mu_{v_j} I_{v_j} \qquad (1) \\
\frac{dN_i}{dt} &= \Lambda_i - \mu_i N_i - \alpha_i I_i
\end{aligned}
$$

Basic Reproductive number R_0

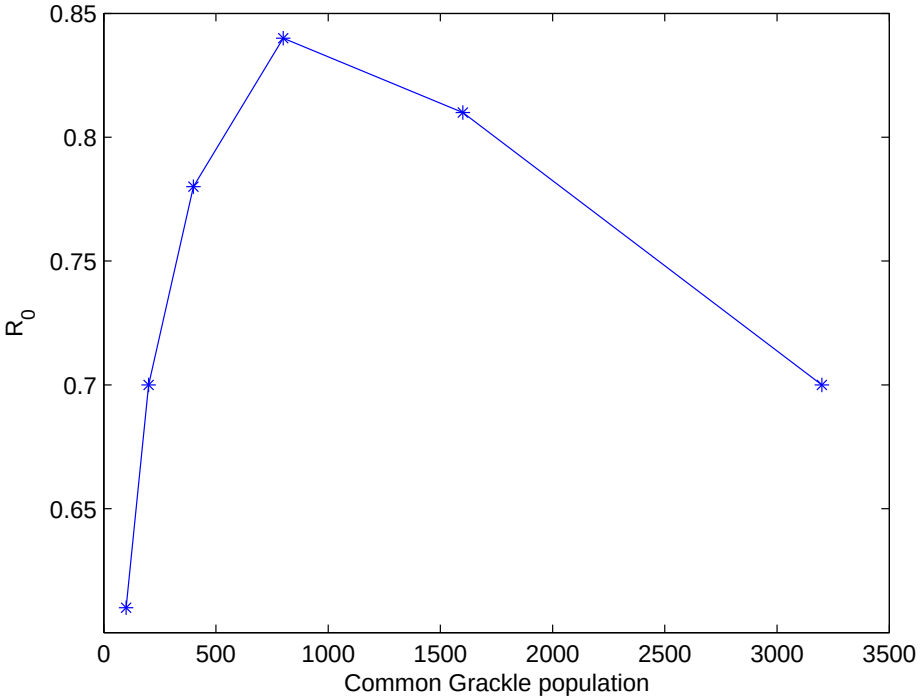
- Spectral ratio of the Next Generation Operator Φ associated to the disease-free equilibrium $S_i = \bar{N}_i$, $I_i = 0$, $i = 1, \ldots n$, $I_{v_j} = 0$, $j = 1, .., m$

$$\Phi = \begin{pmatrix} 0 & G_1 \\ G_2 & 0 \end{pmatrix}$$

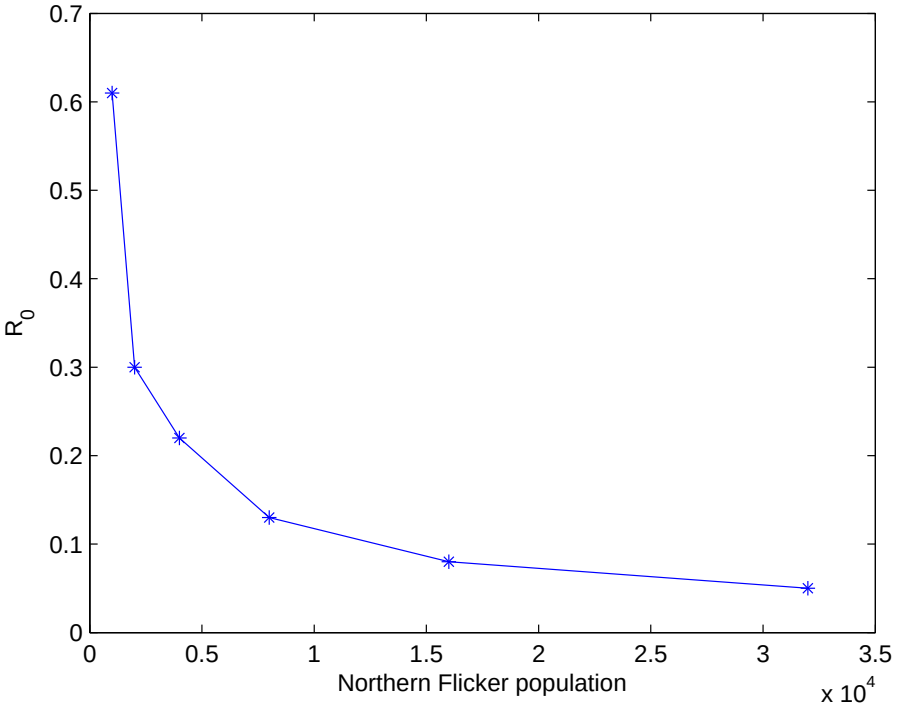
$$G_1 = \frac{1}{\bar{N} N_v} \begin{pmatrix} \frac{b_1 \beta_{11} N_{v_1} \bar{N}_1}{\gamma_1 + \mu_1 + \alpha_1} & . & . & . & \frac{b_m \beta_{1m} N_{v_m} \bar{N}_1}{\gamma_1 + \mu_1 + \alpha_1} \\ \frac{b_1 \beta_{21} N_{v_1} \bar{N}_2}{\gamma_2 + \mu_2 + \alpha_2} & . & . & . & \frac{b_m \beta_{2m} N_{v_m} \bar{N}_2}{\gamma_2 + \mu_2 + \alpha_2} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \frac{b_1 \beta_{n1} N_{v_1} \bar{N}_n}{\gamma_n + \mu_n + \alpha_n} & . & . & . & \frac{b_m \beta_{nm} N_{v_m} \bar{N}_n}{\gamma_n + \mu_n + \alpha_n} \end{pmatrix},$$

$$G_2 = \frac{1}{\bar{N}} \begin{pmatrix} \frac{b_1 \beta_{v_{11}} N_{v_1}}{\mu_{v_1}} & . & . & . & \frac{b_1 \beta_{v_{1n}} N_{v_1}}{\mu_{v_1}} \\ \frac{b_2 \beta_{v_{21}} N_{v_2}}{\mu_{v_2}} & . & . & . & \frac{b_2 \beta_{v_{2n}} N_{v_2}}{\mu_{v_2}} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \frac{b_m \beta_{v_{m1}} N_{v_m}}{\mu_{v_m}} & . & . & . & \frac{b_m \beta_{v_{mn}} N_{v_m}}{\mu_{v_m}} \end{pmatrix},$$

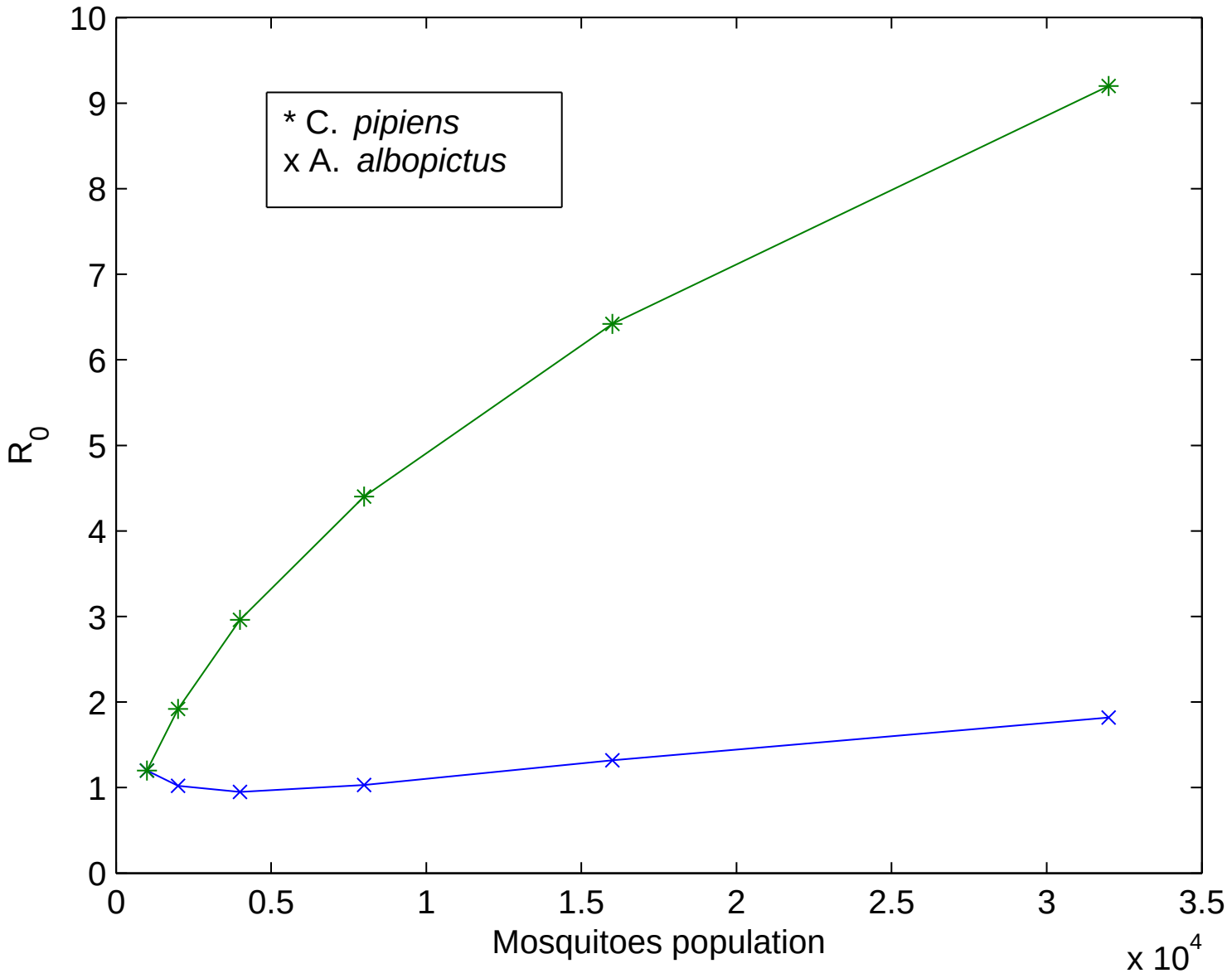
with $\bar{N} = \bar{N}_1 + \cdots \bar{N}_n$, $\bar{N}_i = \Lambda_i / \mu_i$, and $N_v = N_{v_1} + \cdots + N_{v_m}$.



R_0 for different population sizes of Common Grackle species.



R_0 for different population sizes of Northern Flicker species.

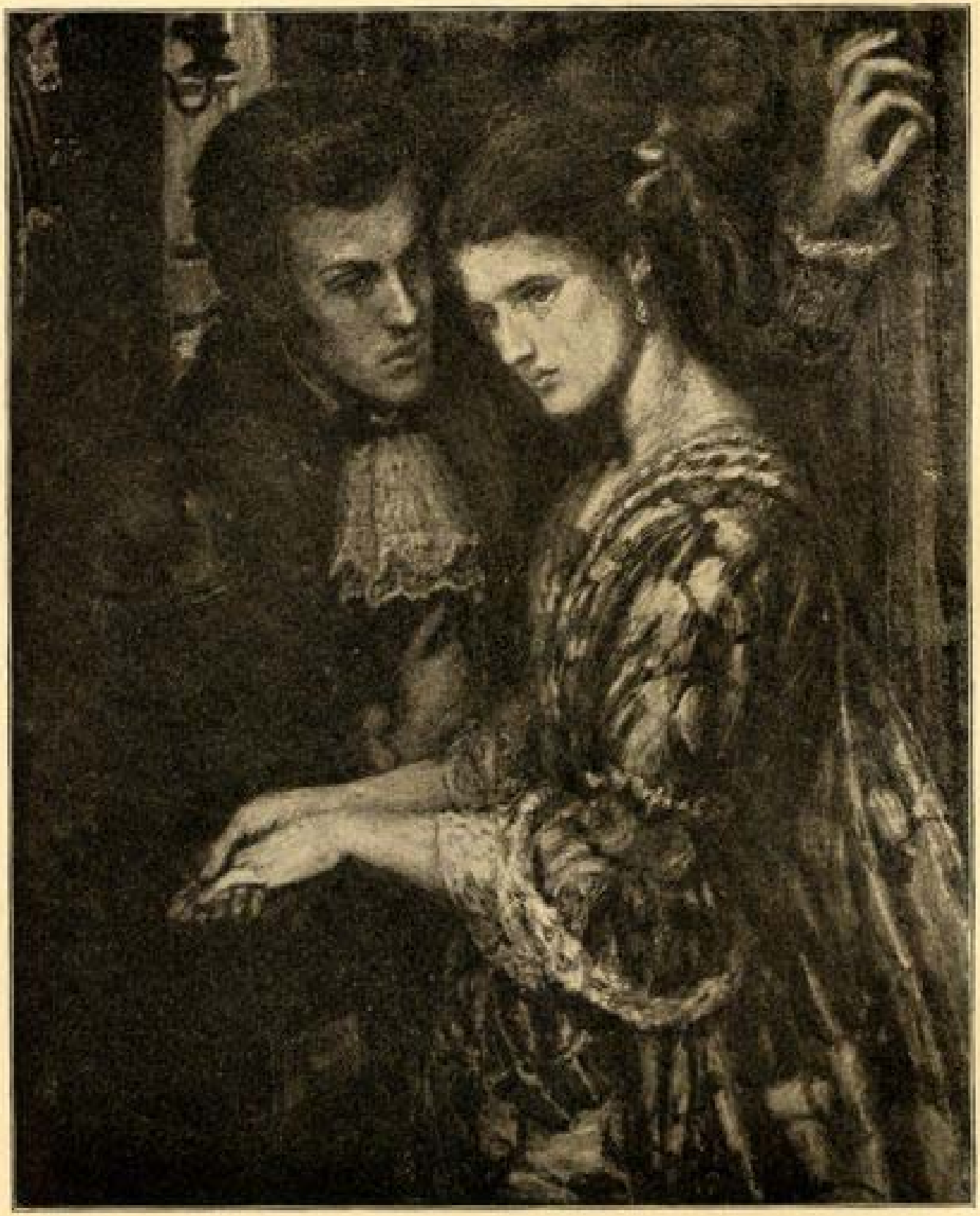

* C. pipiens
x A. albopictus
R0
Mosquitoes population
x 10^4
0
1
2
3
4
5
6
7
8
9
10
0
0.5
1
1.5
2
2.5
3
3.5

Conclusions

- R_0^a and R_0^v estimate the importance of birds and mosquitoes on the prevalence of WNV.
- The competence and abundance of each species determines its role on the maintenance of the disease.
- Species not affected by the disease if isolated, can become infected if they share the space with a large enough competent species.
- The transmission of WNV is sensible to variation on populations density.

Distribution of Cx. *pipiens* in North America. Data taken from http://svs.gsfc.nasa.gov/vs/a000000/a002500/a002565/.

- Culex species that are efficient transmitter of WNV are not common in Mexico.
  http://www.cenave.gob.mx/von/.
- Common mosquitoes species in México (for example, Ae. *aegypti*) are not very competent transmitters .
- A possible explanation of the low prevalence of WNV in Mexico.



WASHING HANDS